Electrostatic Electron Microscopy. I

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This paper, consisting of three parts, describes investigations made with the objective of developing a simplified, practical microscope of the type which yields magnified images of transparent specimens with a resolving power superior to that of the best light microscopes. The first part deals with the general problem of design, including the electron gun and the imaging lenses. A later part will describe a completed instrument embodying many of the results of the investigations.

It is the purpose of this paper to describe investigations made by the authors toward the development of a practical, commercial electron microscope of the type which yields magnified images of transparent specimens with a resolving power superior to that of the best light microscopes. Also, a completed instrument will be described which embodies many of the ideas and principles that were the subject of investigation. The first portion of the paper will not be concerned with any one instrument design but rather will be a general discussion of the problem of design of electrostatic electron microscopes. Although the authors' investigations dealt mainly with the electrostatic method, the magnetic lens type of microscope will occasionally be referred to for comparison purposes.

Many papers have been published both on this continent and abroad on electron microscopy. These papers have been mainly of two kinds: Either a completed instrument has been described, the discussion centering on the instrument's chief performance characteristics and its physical form; or more or less general theoretical discussions have been presented of electron optical systems. Little has been published about what might be called the "engineering" of an electron microscope.[1] While the following is far from a complete dissertation on the optimum design of an electron microscope, an attempt has been made to consider the basic problems which are met in the building of a commercially practical instrument. These problems will be described and a partial solution to many will be given. It is thus hoped that the article will point out the various difficulties of theoretical, experimental, and engineering design nature which arise in microscope development and what might be done about those problems in certain cases.

ELECTROSTATIC *VERSUS* MAGNETIC LENSES

It is known that both electric[2] and magnetic fields[3] can be used to obtain electronic images of high resolution. The authors considered both general types of microscopes as to their suitabilities for a commercial instrument of wide range applicability and also as to the potentialities of each in advancing closer to the ultimate resolving power. The investigations and the sample instrument described in this paper bear out the belief that the electrostatic type is admirably suited to the role of the practical commercial instrument, possibly even more so than the magnetic type because of the inherent simplicity of the former. It is possible by using electrostatic lenses to remove at once one of the most complex components of the electron microscope, the closely regulated power supply. The focal length of a magnetic lens is a function of both the magnetic field strength and the velocity of the electrons being focused by the lens. The magnetic field is derived from a current and the electron velocity from a voltage so both must be very closely regulated or else the focal length will vary too greatly and the image will be blurred. Regulation tolerances of the order of one part in many thousand have been found necessary.[4] In the electrostatic system, the electron velocity and the lens focusing action may be derived from a single high voltage source. The lens focal length in this case is then mainly a function of its physical size and configuration since an increase or a decrease in electron velocity is always accompanied by an increase or decrease in the electric field's focusing action of just such strength as to yield precisely the same electron paths, except for a small relativity effect.

As the lens components in the electrostatic system are of relatively small size and simple configuration, it becomes practical to use multiple lenses held together in a compact enclosure which greatly decreases the volume and area of the chamber to be evacuated.

The authors, however, do not take the view that the investigations reported here constitute a proof that one or another microscope is superior. They feel rather that the field is young and important advances may be expected in each type of instrument, so that investigations of all microscope attacks should be continued and an open-minded attitude should be used in making comparisons between these various attacks.

SCHEMATIC DIAGRAM OF ELECTROSTATIC MICROSCOPE

Figure 1 shows the major electron-optical components of an electrostatic microscope, each of which will be discussed in the remainder of the paper. All parts are at ground potential except the cathode, its shield, and the central electrode of each lens. Two stages of magnification are shown but, as will be explained later, it may prove advantageous to use three or more stages.

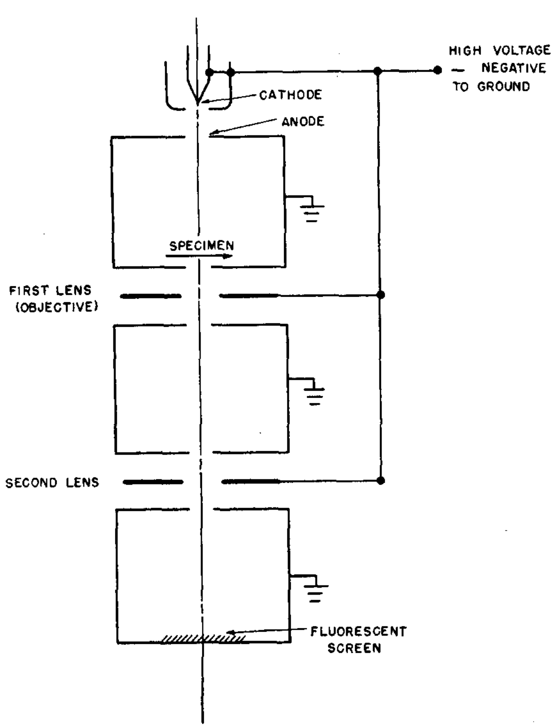

FIG. 1. Diagrammatic sketch of components of an electrostatic microscope.

THE GUN PROBLEM

It is convenient to divide the electron optics of the microscope into two parts: (1) The electron gun, which serves to illuminate the specimen; and (2) the imaging system which magnifies the electron image of the specimen into a larger electron image on the fluorescent screen or the photographic plate. For a simple commercial instrument, it is especially desirable that the gun should be reduced to as few components as possible, consistent with obtaining the resolving power and other performance capabilities desired for the over-all instrument. Since it is possible to operate the imaging system of an electrostatic microscope with only one voltage difference to ground, it is also preferable in the interest of simplicity to avoid those electron guns which require additional voltage sources. Although the authors performed some experiments with multi-voltage guns, a single voltage electron gun was developed which proved quite satisfactory in all respects and this type will be discussed here.

The first basic requirement of the electron gun, regardless of whether the imaging system is magnetic or electrostatic, is that it be capable of producing sufficient current density at the specimen. The cross-sectional area of the electron beam at the specimen and the angle which the electrons make with the axis must also be considered carefully. An important consideration in this regard is the heating of the specimen by the electron beam. It is desirable to obtain as bright a final image as possible for visual observation with a minimum of current striking the specimen. This means that it is disadvantageous to illuminate the specimen with electrons if those electrons, for one reason or another, cannot possibly contribute to the contrast or even appear in the final image at any time. It would be pointless, for example, to irradiate a large part of the specimen if only a small portion on the axis is to be imaged. Also (for the conventional bright field type of illumination) there would appear to be

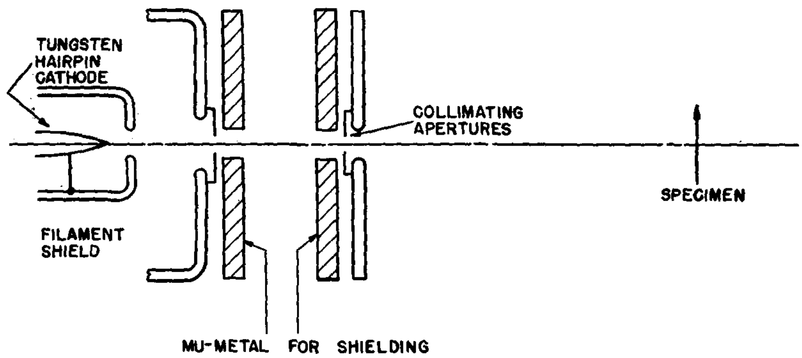

FIG. 2. Arrangement of electrodes in unipotential electron gun.

no reason to aim electrons at the specimen if their incident angle is such that in the absence of deflection they will surely be rejected by the objective lens aperture stop. Of course, the contrast obtained in the final image is the result of the distribution of the electrons arriving there; but every electron should be granted equal opportunity to contribute to the final image of that part of the specimen through which it passes, unless the specimen itself decides otherwise and absorbs it, slows it, or deflects it. Any electron that impinges upon the specimen with so large an angle that in the absence of preferential deflection by the specimen it is certain to be collected at a lens aperture only serves to heat the specimen uselessly.

There is some evidence to indicate that for very thin specimens, the bulk of electrons that come through the specimen do so with their slope virtually unchanged. Thus there is a possibility that the limiting angle of the electron bundle leaving the objective lens can be set mainly by the gun apertures rather than by apertures of the objective. Sometimes this type of limiting is an easier task mechanically because the last aperture in the gun preceding the specimen determines that angle, and that last aperture may be more favorably located than the objective lens stop so as to avoid the need for producing extremely small diameter holes. It is probable that the lens aberrations as well as the particular thickness and material of the specimen that is under examination have an important part in determining whether the gun angle can serve as the effective stop of the system or whether that limit is best determined by the first imaging or objective lens. However, our best results have been obtained with a small gun angle and, although the information along these lines cannot yet be considered complete, indications are that the gun angle has a more easily discernible effect upon the resolving power than does the lens aperture.

The separate condenser lens for focusing electrons on the specimen is often considered a necessary component of an electron microscope. Any gun, since it is a means for doing such focusing, must include in it the focusing properties of a condenser lens. However, it is possible to use an additional condenser lens and electrons coming from a simple gun which might in itself be fairly successful in illuminating the specimen can be additionally focused to give even a more intense concentration on the specimen. Such a separate condenser lens has been used by the authors, but the method of visual observation and photography of the image finally chosen for the instrument to be described later was such that it did not seem necessary to add this additional lens to increase the intensity. Once adequate intensity in the image is obtained additional electron beam current simply serves to damage the specimen and add the possibilities of undesirable space-charge effects in the objective lens.

If, because greater intensity at the specimen is desired, a condenser lens is deemed necessary, it is very easy to add it to the gun design. With proper collimation of the electrons coming from the simple electron gun, a parallel beam can be considered to enter the condenser lens. The rays leaving this lens will focus at a desired point on the axis if the condenser lens is simply located at a distance equal to its focal length ahead of that point.

The simple design of gun indicated diagrammatically in Fig. 2 was chosen by the authors as coming closest to satisfying the general requirements. The proper relation between filament point location, filament-shield aperture diameter, anode-filament spacing, and the spacing and size of the other apertures depends upon a number of factors that will be peculiar to each instrument design, such as the exact location of the imaging lens system and the specimen, the filament life desired (filament life being balanced against intensity of illumination), the accelerating voltage, and the extent of magnetic shielding of the filament current's magnetic field which it is practical to use. In general, it is necessary to adjust the filament location and filament-shield aperture diameter experimentally for each different construction. The information now available on axially symmetric immersion lenses, such as are used in common cathode-ray tubes, is not applicable to the case of the asymmetrical hairpin-type cathode used in the microscope.

An important problem in the electron gun design is overcoming the magnetic field due to filament current, whether it be supplied by d.c. or a.c. The question of magnetic shielding of the instrument will be discussed in more detail later but for the moment it is pointed out that although cross magnetic fields (those components perpendicular to the axis of the instrument) can be detected by the resolution variation with azimuthal angle about the axis, the axial field causes a bad effect not so easily traced to the filament current.

CONSIDERATIONS AFFECTING CHOICE OF MAGNIFYING LENSES

The electron lenses which form the imaging system of the electron microscope must possess the following general characteristics:

(1) The focal length should be positive and short without resorting to minute dimensions.

(2) If the lens is to be the objective lens, then the "in-focus" position of the specimen must be external to the high field region of the lens. As a matter of fact, it will generally be absolutely necessary to insure that the specimen is in a region which is, practically speaking, entirely free from electric fields.

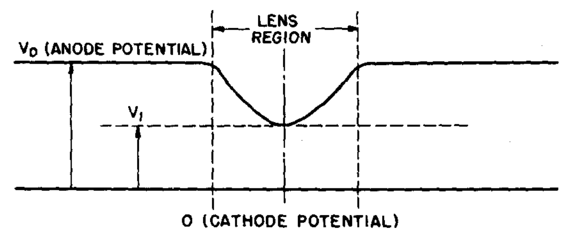

FIG. 4. Axial potential distribution of unipotential lens.

(3) The electrodes should have such physical form that the insulation problem will be minimized and field emission at the operating voltage cannot take place.

(4) The lens should preferably be symmetrical in general physical form with a single potential to ground, namely, that of the cathode, sufficing to energize the lens.

(5) There should not be an excessive drop in axial voltage in the central section of the lens because, as will be shown later, both the stray magnetic field and chromatic aberration effects increase with this drop and they may easily become excessive.

(6) The lens should be readily manufacturable to the required precision.

(7) Spherical aberrations and other purely geometric defects should be low enough to yield the desired resolving power without necessitating the use of impractically small lens stops.

It is quite evident that with all of these requirements to consider, it is not a simple matter to arrive at the optimum lens design. However, certain of these seven factors may be taken as basic or essential and others may be regarded as ideals toward which one should work. For example, the requirement that the lens electrodes must be either at ground (anode) or cathode potential may be taken as a fundamental point. Then, the schematic diagram of Fig. 3 shows

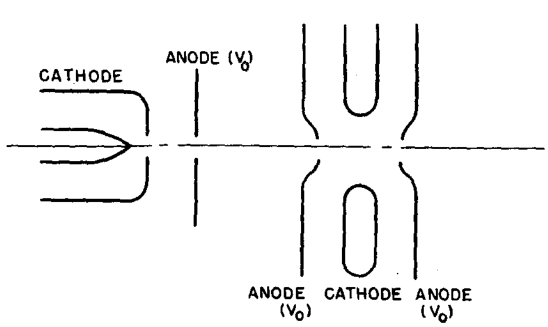

FIG. 3. General form of unipotential lens.

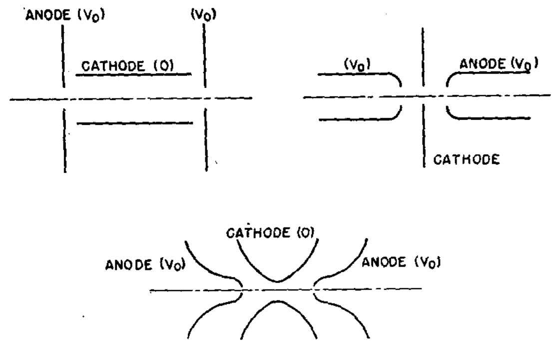

FIG. 5. Unipotential lens configurations.

the general form which the lens might have. Shown in this figure are three axially symmetric apertured conductors, the two similar electrodes being at ground or anode potential and the central electrode being at cathode potential. With such an arrangement the potential distribution along the axis will take the general form indicated by the diagram of Fig. 4. Here the anode-cathode voltage is called V_0 and the minimum voltage at the lens center V_1. The use of small axial holes in the grounded electrodes will insure that the electric field will die off rapidly with distance outside the lens region where, in the case of the objective, the specimen will have to be placed. These holes may also be the lens "stops." What is next needed is to determine more precisely the shapes and spacings of these electrodes and the way in which they are to be mounted so that the requirements stated above will be satisfied. Plane and cylinder combinations or more complex curves (Fig. 5) may be used. The problem is one of synthesis, and hit-or-miss experimentation cannot be relied upon exclusively to yield optimum results.

Now, the focal properties of a lens are completely determined when the distribution of potential along the axis is specified. Figure 4 gives the general form of potential distribution which the lens must have, but the particular distributions that are possible are infinite. It is accordingly desirable to know what is to be the preferred distribution by computing focal properties for a variety of theoretical distributions. A convenient way to attack this problem is to choose a simple mathematical expression for the axial potential inside the lens region that contains parameters which, if varied, will yield a variety of lens voltage distributions. In this way it would be expected that certain basic relations would appear that would serve as guides to the choice of the most practical lens design.

Now, there are several simple ways in which the potential can vary, some of the simplest being: (1) The axial potential can increase linearly with axial distance from the lens center until it reaches anode potential V_0, at the entrance and exit of the lens (Fig. 6). This distribution is closely approached in a physically realizable lens consisting of three plates at potentials V_0, V_1, and V_0 equally spaced and possessing very small axial holes. (2) The axial potential may vary sinusoidally with distance according to the function

$$V \propto 1 - c \cos \omega z, \qquad (1)$$

where z is the axial distance from the lens center. By varying the two parameters, c and ω, a large variety of interior lens distributions are obtained (Fig. 7). (3) The axial potential may vary exponentially with axial distance from the lens center. Since we are restricting the discussion to symmetrical lenses, the simplest exponential type of variation probably is:

$$V \propto A\left[\frac{\epsilon^{\tau z}+\epsilon^{-\tau z}}{2}\right] = A \cosh \tau z. \qquad (2)$$

The two parameters A and τ provide further

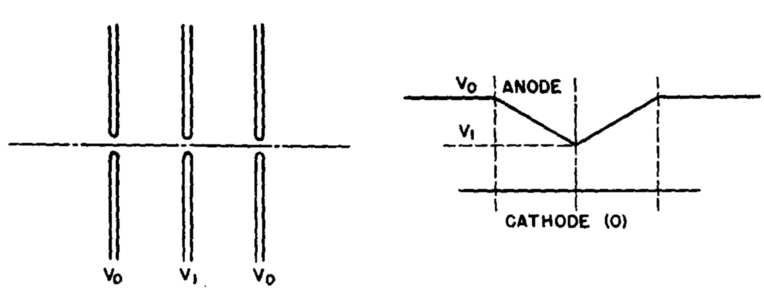

FIG. 6. Lens with nearly uniform fields between electrodes.

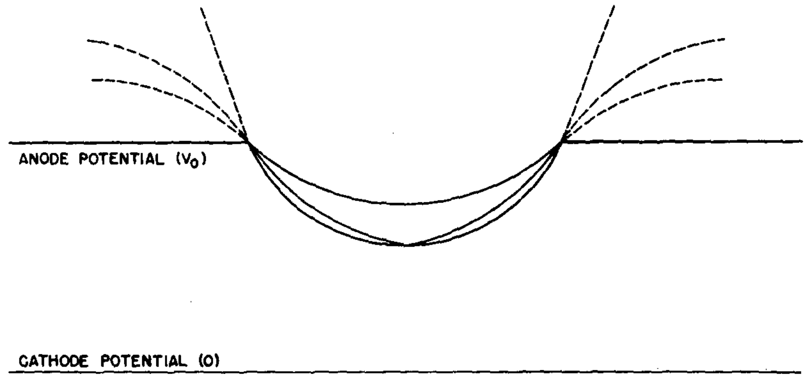

FIG. 7. Axial potential distributions in the cosine-Bessel lens.

wide varieties in distributions. The two general lenses corresponding to Eqs. (1) and (2) are physically realizable. From the general theory of electrostatics,[5] it is found that there are solutions to Laplace's equation, which all field distributions must satisfy, that correspond to the axial distributions of Eqs. (1) and (2). These are:

$$\Phi \propto 1 - cJ_0(i\omega r) \cos \omega z \tag{3}$$

and

$$\Phi \propto A \cosh \tau z J_0(\tau r), \tag{4}$$

where J_0 is the Bessel function of zero order and r is radial distance from the axis. With the equations for potential known, one has only to equate Φ to V_0 to determine the equation for the surface of the two anode-voltage electrodes, and to equate Φ to zero to obtain the form of the cathode potential electrode. Two examples are shown in Fig. 8.

METHODS FOR COMPUTING FIRST-ORDER FOCAL PROPERTIES OF LENSES

There are a number of special methods which apply particularly well to the type of lens of interest in an electrostatic microscope. For one thing, the boundary apertures will always be relatively small in diameter as compared to their focal length so that the aperture lenses thus formed may be treated as thin lenses superimposed on the entrance and exit of the lens proper. These two lens actions at the apertures are divergent;[6] their focal lengths are easily computed[6] after an estimate is made of the field existing inside the lens near the aperture. (The field on the other side is taken as zero.) The center of the lens is a relatively highly convergent system. This central lens region can be treated by thick lens formulas[6] or as a combination in itself of three thin lenses. In the case of the "uniform field" potential distribution of Fig. 6,

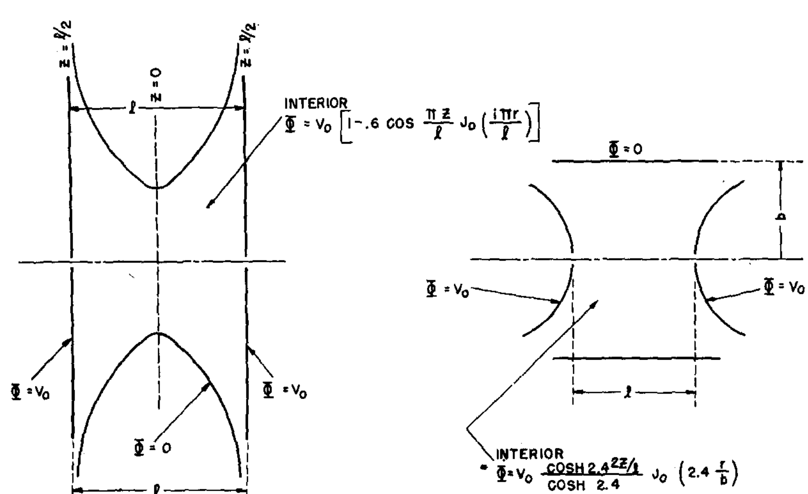

FIG. 8. Examples of cosine-Bessel and cosh-Bessel lens electrode configurations.

the central of these three lenses is regarded as simply another aperture lens, separated from the entrance and exit apertures by uniform fields whose effects are easily computed.[6]

Because of the symmetry of the problem, it is often easier to trace two particular rays from the center of the lens to the exit, rather than to start the analysis from the entrance and carry it through to the exit. One of these two specially chosen rays (Fig. 9) passes the center of the lens region with a slope toward the axis of zero at an arbitrary distance from the axis; the other has an arbitrary slope at the lens center and it crosses the axis there. The complete path of the first-mentioned "parallel" ray theoretically must, from symmetry, intersect the axis on both the entrance and exit of the lens at the same distance to the lens center. From simple geometrical optics, the distance from either of these intersection points to the corresponding principal plane must equal twice the focal length of the lens. If the principal plane location were known, then the focal length would follow from the location of these intersection points. The second (central) ray, from symmetry, must enter and leave the lens region making the same angle with the axis. If its path on leaving or entering is extended to intersect the axis, this intersection point is a principal point of the lens. Thus from these two "half-paths," the focal length and principal point location of the lens are obtained. We do not then have to make computations of the complete lens region, but only half of it.

FIRST-ORDER PROPERTIES OF PRACTICAL LENSES

By varying the parameters of the three potential distribution functions which have been described, the uniform field, cosine, and cosh, data which are applicable to a practical electron microscope were obtained. For the objective lens, the object position must fall outside the lens so lenses whose focal points fall at a distance less than $l/2$ from the lens center are eligible only for the second and later lenses. The chief criterion of a lens for focal lengths longer than $l/2$ was found to be the voltage drop, V_0-V_1, in the lens. For the same voltage drop, up to about

$$(V_0-V_1)/V_0=0.5,$$

all variations of the three lens distributions considered yielded very close to the same focal lengths. Furthermore, the principal plane locations were found to be very close to the center of the lens for all three cases. This latter was especially true of the "uniform field" lens and the cosine-Bessel lens with the principal planes of the cosh-Bessel lens lying within about 5 percent of the lens length from the lens center. The critical focal length ($f=l/2$) occurred for a voltage drop of between 40 and 50 percent of the cathode-anode voltage.

Accordingly, as far as the objective lens is concerned, there seems little reason to the authors to seek out special voltage distributions which might offer shorter focal lengths while still allowing the object to be placed outside the lens region. For lenses other than the objective lens, the study must be regarded as much farther from complete, for with the lens voltage drop exceeding around 50 percent of the cathode-anode voltage, the principal planes move away

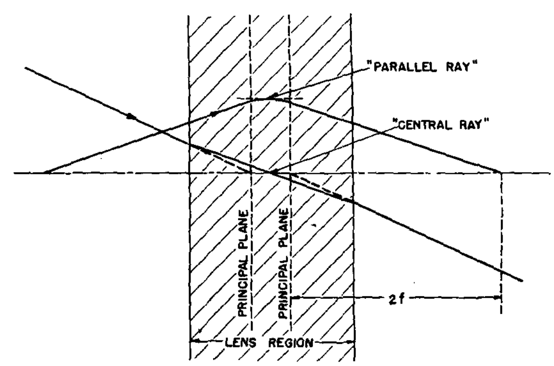

FIG. 9. Rays in a symmetrical lens.

from the lens center and it becomes less accurate to say that the focal length depends almost entirely upon the voltage drop. It is quite possible that lenses other than the objective might be built with exceedingly smaller focal length to lens length ratios than have been used thus far by the authors. However, there are manufacturing simplifications in building all lenses the same. Also, as has already been mentioned, the chromatic aberration of an electrostatic lens and the effect of stray magnetic fields may become objectionably large if large voltage drops are used, so that both of these factors constitute further limits to the decreasing of focal lengths.

With the knowledge that varying lens poten-

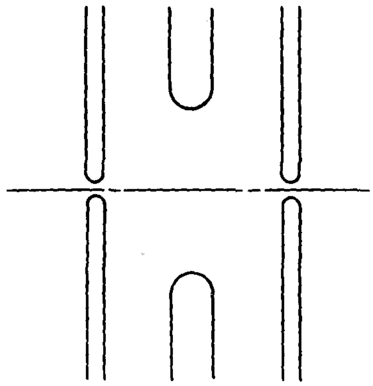

FIG. 10. Practical lens electrode configurations.

tial distributions, unless they depart quite substantially from that pictured in Fig. 6, will result in no huge improvement in the ratio of focal length to over-all lens dimensions for the objective lens, one is then left with the second-order properties to consider in choosing a lens configuration as well as the very practical problems having to do with insulation and possible field emission from the electrode surfaces. From the latter standpoint, further study and experiment indicated that field emission need not occur at voltages of 25 kv to 50 kv so long as no attempt is made to shrink dimensions below those yielding a focal length of about one centimeter. The most serious practical problem is that of insulating the central electrode at its support. The insulator must, of course, be capable of being machined to hold the lens in the required precision of alignment and the shape and type of material must satisfy vacuum requirements. Both of these factors may in turn affect the choice of the lens electrode configurations. Consideration of these factors and a number of experiments finally resulted in the choice of lens having the general configuration shown in Fig. 10. Figure 11 shows in a general way how the focal length varies with the chief lens dimensions.

BAND SPREAD OF FOCAL LENGTHS DUE TO ABERRATIONS

We shall consider next the matter of certain second-order effects or aberrations. Electron-lens aberrations, particularly chromatic and spherical, have been studied often,[8] but the discussion of the problem (particularly as applied to electron microscope design) is not likely to be complete for some time. What is needed is an expression for resolving power as limited by these aberrations in terms of the design parameters of the microscope-lens system: (1) lens configuration, regardless of its physical size, i.e., lens type; (2) focal length, which for a given geometry fixes the lens dimensions; (3) number of stages for a given magnification, or object and image distances; (4) lens-stop radius. It is conceivable, for example, that lens number 1 may have inherently less aberration than lens number 2 judged by keeping all design parameters constant (except, of course, the form of the electrodes). But lens number 2, simply by virtue of its different geometry, may make practical a superior choice of the other parameters (as, for example, physical dimensions for a given sparking voltage) and the final lens design for a particular microscope application will finally prove the number 2 lens to be the one with the higher resolving power.

If an expression for the band spread of focal lengths for electrons ranging from those extremely close to the axis to those limited by the lens opening is derived, then a convenient expression for the resolving power in terms of the microscope design parameters can be easily set up. Figure 12 shows electrons leaving the object at P and arriving at the image which is spread between Q_1 and Q_2, a distance Δq apart, due to the spread in focal length, Δf, between electrons near and far from the axis. If p and q are object and image distances to the proper principal planes, as shown, then elementary

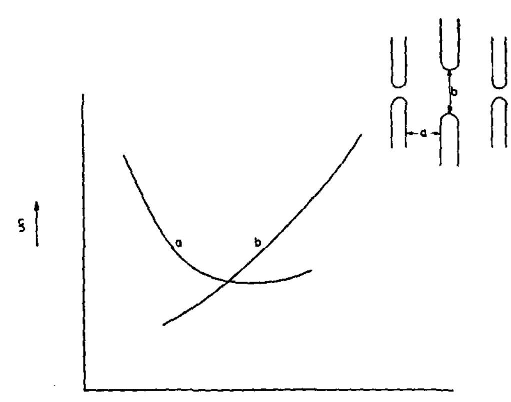

FIG. 11. Dependence of focal length on lens dimensions.

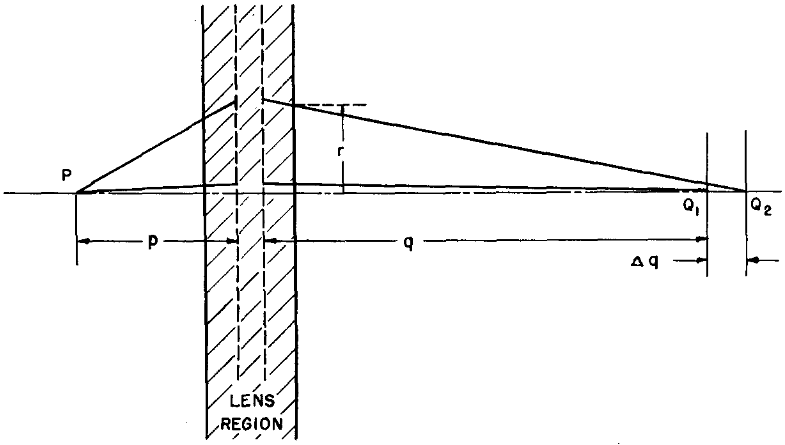

FIG. 12. Variation of image distance with separation from axis at lens exit.

optics states that

$$1/p+1/q=1/f,$$
$$1/p+1/(q+\Delta q)=1/(f+\Delta f).$$

Subtracting the two equations and simplifying gives the approximate result

$$\Delta q=q^2/f^2\Delta f.$$

From this longitudinal aberration the radius of the circle of aberration in an image plane placed halfway between the extreme positions, Q_1 and Q_2, will be about

$$r_q=(q/2)(\Delta f/f^2)r.$$

There may be some question as to just where between Q_1 and Q_2 the image plane should be placed. The optimum location will vary in general with the distribution of electrons in angle and the type of aberration. The above equation should give correct order of magnitude in any case.

Dividing by the magnification we obtain an approximate expression for the resolution (diffuse circle radius in the object plane)

$$\delta=(p/2)(\Delta f/f^2)r. \qquad (5)$$

We are now ready to consider the magnitude of Δf for various lenses and types of lens aberrations.

SPHERICAL ABERRATION

In the appendix to this paper, it is shown that the spread in focal lengths due to spherical aberration may be expressed in the following form:

$$\Delta f=Sr^2/f, \qquad (6)$$

where r^2 is the radius of the stop at the lens exit, f is the lens focal length, and S is a dimensionless constant characteristic of the lens electrode configuration. For a given lens "type," S is fixed—then choice of any dimension determines f or choice of f determines the dimensions. Step-by-step computations for S, as sketched in Appendix A, were made for two lens potential distributions, one a cosine-Bessel lens and the other a cosh-Bessel lens. S was found to be of the order of magnitude of 5 for the cosine-Bessel lens and much less for the cosh-Bessel lens. Let us take this figure of 5 as a trial value to probe the possibilities of reaching high resolving power with the unipotential lens, always with the restriction that this is only one special case and should not be looked at as necessarily indicative of the best lens that can be designed. From Eqs. (5) and (6)

$$\delta=(pS/2f^3)r^3. \qquad (7)$$

For an objective lens whose focal length is 1 cm and which images at a sufficient distance to make p approximately equal to f, we find for the lens of $S=5$,

$$\delta=2.5r^3,$$

so that an aperture radius of about 5×10^{-3} cm would be small enough to insure 30A resolving power.

In computing S for the cosh or cosine potential distributions, the entrance and exit apertures were assumed to be small in diameter compared to lens length and, though the departure from exact cosine or cosh distribution due to these apertures was not accurately known, it was possible to assign maximum values to the aberrations contributed by them. The coefficient B of Eq. (69) (appendix) is a measure of the spherical aberration and this equation may be looked at as stating that there is a limit to the value which B can attain for a lens of a given length. In particular, for the case of a very thin aperture lens (length of lens region small compared to its focal length), the voltage may be assumed substantially constant, and an approximate integration of (17) may be made. This point does not warrant further discussion here. It will probably be sufficient to state that for the two lenses considered the maximum values obtained for the additional spherical aberrations contributed by the apertures were computed to be too low to alter the order of magnitude of the figure previously given for S.

CHROMATIC ABERRATION

Formulas have been derived previously for chromatic aberration of electrostatic lenses.[8] A common formula is

$$\Delta f = (\Delta V/V)f, \tag{8}$$

where V is the voltage of the average electron and ΔV is the variation in voltage of electrons passing through the lens. It is quite apparent that this simple equation is quite inadequate for microscope lenses in which the voltage drop in the lens for all electrons is an appreciable fraction of the anode-cathode voltage. For this case it is convenient to think of the lens as consisting of a series of thin lenses, those near the entrance and exit being divergent and those near the lens center being convergent. The convergent action must, of course, be stronger than the divergent effects to yield a net convergent lens. But this converging action takes place at the lowest electron speeds. Thus in Eq. (8) one should use V_1 rather than V_0 for V for that portion of the lens. It was this effect that was mentioned previously as a limiting factor in attaining short focal lengths by utilizing large voltage drops.

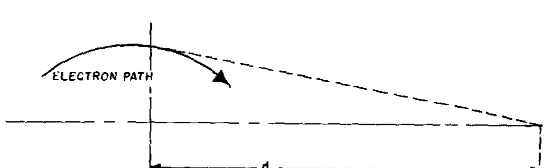

FIG. 13. Focal distance of electron.

For lenses in which the drop is near one-half V_0, a study of the cumulative action of the thin lenses assumed to make up the complete lens indicated that

$$\Delta f = (2\Delta V/V)f \tag{9}$$

will be a reasonably good approximation. Combining this equation with Eq. (5) yields

$$\delta = p/f(\Delta V/V)r.$$

The magnetic lens does not affect the magnitude of the velocity of the electrons. There is no

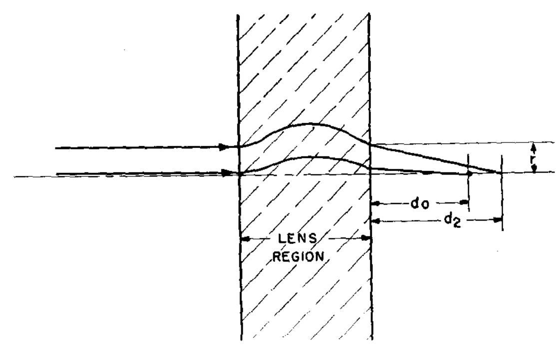

FIG. 14. Variation of focal distance for principal rays.

occasion then to add a factor, such as the 2 of Eq. (9) to the aberration. For the same focal length and aperture radius, a magnetic lens may accordingly be expected to have less chromatic aberration than the electrostatic lens. The effect of this difference may be insignificant or important depending upon the complete instrument design.

APPENDIX—SPREAD OF FOCAL LENGTHS DUE TO SPHERICAL ABERRATION

In the paper already referred to, Gray describes an electron's motion in an axially symmetric electrostatic field by specifying an instantaneous "focal distance" for each point of its path (Fig. 13). This distance d is the distance measured along the axis from the electron's position to the point on the axis toward which, or away from which, it is moving. With this definition and the usual equations of motion, the following approximate relation is obtained:

$$\frac{1}{d} = \frac{-A + Br^2 - \cdots}{(2V)^{\frac{1}{2}}}, \tag{69}$$

where the coefficients A and B are determined by

$$(2V)^{\frac{1}{2}}A'+A^2=-\frac{V''}{2}, \tag{16}$$

$$(2V)^{\frac{1}{2}}B'+4AB=\frac{V''''}{16}-\frac{(A')^2}{2}, \tag{17}$$

the primes indicating differentiation with respect to the axial coordinate z and the equation numbers being Gray's.

Let us first consider the case of no aberration or

$$\frac{1}{d}=-\frac{A}{(2V)^{\frac{1}{2}}}, \tag{1a}$$

where A is determined completely by Eq. (16). If we choose a point on the axis in front of a lens of given potential distribution as a source of electrons (Fig. 14), we can plot any given electron's path through the lens by any of the well-known step-by-step methods.[7] One way, of course, is to start with d_1 and consequently [from Eq. (1a)] A_1 as known quantities at the lens entrance z_1, and compute A' and A step-by-step through the lens from Eq. (16). In this way we end up with A_2 at the lens exit z_2 and compute d_2. It is the variation of d_2 with r when B is included that gives the first approximation to the spherical aberration. If B_2, the value of B at the exit is known, the aberration can be computed, at least as to order of magnitude. Now B_1, the value of B at the entrance z_2 is zero, since the electrons all are specified to come from a single point on the axis, regardless of what their radial displacement is when entering the lens. Since the values of A and A' are known approximately from the first-order electron trajectory, we can now use Eq. (17) for a step-by-step computation of B.

An advantage of this technique of investigating spherical aberration over most other methods which have been described[8] is that it is possible to see how the spherical aberration is a result of distributed contributions from different parts of the lens region. The instantaneous value of the coefficient B is at any position along the lens a measure of the accumulated spherical aberration up to that point. It starts out with an initial value of zero. For a first approximation to freedom from spherical aberration, B should be zero or negligibly small at the lens exit. Notice that it is not necessary that it be zero at every point in the lens, so that the possibility of compensation between various portions of the lens is provided for. Taking a cue from light optics, we would expect that such compensation of the first disturbing term in the series [Eq. (69)] offers a possibility for improved lens systems.

With B_2 known, it is obviously possible to express the radius of the diffuse image of a single axial point. This Gray does in his Eq. (74). Such an expression would not tell the whole story with regard to the spherical aberration of a lens for use in an electron microscope. For this application, it is convenient to have an expression for the spread of focal lengths due to spherical aberration. There will be a band of focal lengths ranging from f to $f+\Delta f$, for the bundle of electrons leaving the lens at a distance from the axis ranging from zero to a radius of r. We choose an electron whose d at the lens entrance is $-\infty$, and whose d at the exit is therefore such that the electron crosses the axis at a focal point (Fig. 14). In the absence of spherical aberration B_2 is zero; we denote the exit d for this condition by d_0. Then

$$\frac{1}{d_0}-\frac{1}{d_2}=\frac{B_2r^2}{(2V_0)^{\frac{1}{2}}} \tag{2a}$$

or, approximately

$$\frac{d_2-d_0}{d_0^2}=\frac{B_2r^2}{(2V_0)^{\frac{1}{2}}}.$$

Since $d_2-d_0=\Delta f$

$$\Delta f=\frac{d_0^2B_2r^2}{(2V_0)^{\frac{1}{2}}}. \tag{3a}$$

For any given lens, Δf varies with the square of the lens stop radius. We now imagine the whole diagram of Fig. 14 to be magnified by some scale factor l, which increases all lens dimensions and the initial distance of the extreme ray from the axis in the same proportion. It is quite apparent that if the new electron paths through the new lens are to continue to be described by the equations we have written, then

$$l=\frac{d^*}{d}=\frac{A}{A^*}=\frac{Br^2}{B^*r^{*2}}$$

or

$$\frac{B^*}{B}=\frac{r^2}{lr^{*2}},$$

where the asterisk indicates the quantities after the scale change. Since r^{*2} is actually l^2r^2, we learn in this way that for a given lens geometry, B varies with inverse cube of any lens dimension. Since the focal distances and focal lengths are each proportional to the linear physical dimensions for any given lens geometry, and since V_0 is a constant independent of lens design, we may substitute in Eq. (3a)

$$d_0^2\propto f^2;\quad \frac{B_2}{(2V_0)^{\frac{1}{2}}}\propto\frac{1}{f^3} \tag{4a}$$

to obtain finally

$$\Delta f=Sr^2/f, \tag{5a}$$

where S is an aberration constant, characteristic of the particular type of lens. It is dimensionless, a function only of the lens' geometrical configuration. Its numerical magnitude can be found once B_2 has been found, of course, by comparing Eqs. (3a) and (4a).

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